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EVALUATION OF SUPERCONDUCTING DIPOLES

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August 16, 1979

A superconducting dipole can be approximated by an axial cylindrical sheet of current at radius r with a cosine azimuthal distribution $I_o \cos\theta$. This current distribution produces a pure dipole field

$$B_y = B_o = 2\pi I_o.$$

Deviations from this ideal distribution can be written as a summation of error multipole currents $I_n \cos(n+1)\theta$ for $n = 1,2,3\cdots$. Each of these currents produces a multipole field with spatial derivatives

$$B^{(n)} \equiv \frac{d^n B_y}{dx^n} = 2\pi I_n \frac{n!}{r^n} .$$

The multipole field coefficients b_n defined by

$$B_y(x) = B_o(1+b_1x+b_2x^2+b_3x^3+\ldots)$$

are then

$$b_n = \frac{1}{n!}\frac{B^{(n)}}{B_o} = \frac{I_n}{I_o}\frac{1}{r^n} .$$

We define the "normalized" multipole coefficients as $r^n b_n$ which are then equal to the relative multipole currents I_n/I_o. If the multipole current is rotated as $I_n \cos(n+1)(\theta-\theta_n)$ there will exist in addition, skew coefficients a_n defined by

$$B_x(x) = B_o(a_1x+a_2x^2+a_3x^3+\cdots)$$

and the corresponding "normalized" skew coefficients $r^n a_n$.

Multipole currents arise from two sources.

A. A practical design is never ideal. Intrinsic to a design, certain multipole currents may be present. These "intrinsic multipole coefficients", generally b_n with even n, are usually rather large and must be compensated by correction magnets, at least for low orders.

B. Imperfections in construction will yield non-zero values for all a_n and b_n. These are random. The standard deviations of the normalized coefficients $r^n a_n$ and $r^n b_n$ give the "scatter" of the relative error multipole currents I_n/I_o which taken together, constitute a measure of the errors in placement of the coil conductors.

We can, thus, make the following interpretations:

Field Qualities as related to beam dynamics are given by the unnormalized coefficients a_n and b_n. Tolerances on these coefficients are set by beam dynamics considerations.

Construction Accuracies, specifically, accuracies in the placement of coil conductors and magnetic surfaces are given by the normalized coefficients $r^n a_n$ and $r^n b_n$.

With these understandings, two features become immediately obvious.

1. Given the highest achievable accuracy in conductor placement, the only way to further improve the field quality is to increase the coil aperture radius r.

2. Since the coil is generally composed of rather fine-grained conductors, one can expect sizeable error multipole currents

I_n/I_o up to rather high n. The only way to make a_n and b_n fall off with increasing n (required by beam dynamics considerations) is, again, to use a large coil aperture.

As illustration we tabulate here the measured data on some Fermilab Energy Saver and Brookhaven Isabelle dipoles.

Energy Saver Dipoles (16 dipoles No. 200 - 221)

(r = 1.5 in)

	Field Quality		Multipole current I_n/I_o	
n	$a_n(10^{-4}in^{-n})$	$b_n(10^{-4}in^{-n})$	$r^n a_n(10^{-4})$	$r^n b_n(10^{-4})$
1	-0.15±4.22	-0.89±1.67	-0.2±6.3	-1.3±2.5
2	-1.07±1.48	-4.52±3.15	-2.4±3.3	-10.2±7.1
3	-0.87±2.11	-0.22±0.80	-2.9±7.1	-0.7±2.7
4	-0.42±0.46	0.63±2.51	-2.1±2.3	3.2±12.7
5	-0.28±0.70	-0.35±0.62	-2.1±5.3	-2.6±4.7
6	-0.06±0.50	6.16±0.48	-0.7±5.7	70.1±5.5
7	0.29±0.45	-0.28±0.32	4.9±7.7	4.7±5.4
8	0.20±0.41	-17.17±0.57	5.0±10.6	-440±15
9	0.73±0.41	0.02±0.62	28.2±15.8	0.7±23.8
10	-0.19±0.40	5.58±0.39	-11.1±23.1	322±22
11	-0.39±0.48	-0.13±0.32	-34.1±41.8	-11.2±28.0
12	-0.04±0.39	-1.46±0.34	-5.6±50.5	-190±43
13	0.06±0.28	-0.04±0.31	12.2±55.1	-8.0±61.1
14	0.03±0.28	0.09±0.22	9.8±82.0	26.1±62.8

Isabelle Dipoles (5 dipoles MK VI - XIV)

(r = 6.08 cm = 2.39 in)

	Field quality		Multipole current I_n/I_o	
n	$a_n(10^{-4}in^{-n})$	$b_n(10^{-4}in^{-n})$	$r^n a_n(10^{-4})$	$r^n b_n(10^{-4})$
1	1.75±5.08	0.77±1.10	4.2±12.1	1.8±2.6
2	-0.68±0.74	-3.63±3.44	-3.9±4.2	-20.8±19.7
3	0.68±0.86	-0.030±0.303	9.3±11.8	-0.4±4.2
4	0.019±0.134	0.214±0.212	0.6±4.4	7.0±7.0
5	-0.081±0.097	0.050±0.038	-6.4±7.6	3.9±3.0

[These are computed from data given in the paper by E. Bleser et al, IEEE Trans. on Nucl. Sci., p. 3903, Vol. NS-26, No. 3, June 1979. Data from the new bigger aperture (r = 6.55 cm) dipoles B0001 to B0006 built by Westinghouse have not yet been completely analyzed.]

From these tables we can conclude:

I. The intrinsic coefficients (b_n with even n) are large as expected. Their design values for the Energy Saver dipoles are

	$b_n(10^{-4}in^{-n})$	
n	design	measured
2	0.04	-4.52±3.15
4	1.04	0.63±2.51
6	4.44	6.16±0.48
8	-12.09	-17.17±0.57
10	3.63	5.58±0.39
12	-0.82	-1.46±0.34
14	0.07	0.09±0.22

where the measured values are repeated for comparison. The sextupole term (n = 2) has been adjusted in construction and hence, is not expected to agree with design. However, the agreement of all other coefficients is also not very good. The differences between measured and design values generally fall outside of the standard deviations which are themselves already larger than those of the neighboring non-intrinsic coefficients. The same general feature was observed also for Isabelle dipoles.

II. Imperfection multipole currents (normalized coefficients $r^n a_n$ for all n and $r^n b_n$ for odd n) are generally zero within one standard deviation as they should be. Their "scatter" is about the same ($\sim\pm 6\times10^{-4}$ for all $n\lesssim 7$) for both the Energy Saver and the Isabelle dipoles indicating that the absolute accuracies achieved in conductor placement are about the same for both designs. This is a strong indication that we may have reached some kind of limit in practically attainable accuracy. For Energy Saver dipoles the "scatter" increases steadily from n = 7 to n = 14 by more than an order of magnitude. We expect that the same is true for Isabelle dipoles.

III. Field qualities as exhibited by the unnormalized imperfection coefficients (a_n for all n and b_n for odd n) are better for Isabelle dipoles because of the larger coil aperture.

a. For $n\lesssim 5$ the measured standard deviations and the prescribed tolerances (in parentheses) of a_n and b_n are the following.

Energy Doubler Dipoles

n	$a_n(10^{-4}in^{-n})$	$b_n(10^{-4}in^{-n})$
1	±4.22 (±2.5)	±1.67 (±2.5)
2	±1.48 (±2.0)	±3.15 (±6.0)
3	±2.11 (±2.0)	±0.80 (±2.0)
4	±0.46 (±2.0)	±2.51 (±2.0)
5	±0.70	±0.62

Isabelle Dipoles

n	$a_n(10^{-4}in^{-n})$	$b_n(10^{-4}in^{-n})$
1	±5.08 (±2.03)	±1.10 (±2.03)
2	±0.74 (±0.52)	±3.44 (±0.52)
3	±0.86 (±0.25)	±0.30 (±0.25)
4	±0.134 (±0.12)	±0.212 (±0.12)
5	±0.097 (±0.05)	±0.038 (±0.05)

We see that the tolerances are in both cases more-or-less met by the measured values, but the tolerances specified are much tighter for Isabelle. Whether the looser tolerances for Energy Doubler are adequate is not the subject of this paper, but should certainly be studied in detail.

b. Multipole coefficients with $n>5$ were not measured for Isabelle dipoles. But if we assume the same conductor placement accuracy (same standard deviations for $r^n a_n$ and $r^n b_n$) the standard deviations of a_n and b_n of the Energy Saver dipoles when scaled to Isabelle dipole aperture should give those for the Isabelle dipoles.

n	$a_n (10^{-4} in^{-n})$	$b_n (10^{-4} in^{-n})$
1	±2.65 (±5.08)	±1.05 (±1.10)
2	±0.58 (±0.74)	±1.24 (±3.44)
3	±0.52 (±0.86)	±0.20 (±0.30)
4	±0.071 (±0.134)	±0.387 (±0.212)
5	±0.068 (±0.097)	±0.060 (±0.038)
6	±0.030	±0.029
7	±0.017	±0.012
8	±0.0098	±0.0136
9	±0.0061	±0.0093
10	±0.0038	±0.0037
11	±0.0028	±0.0019
12	±0.0014	±0.0013
13	±0.00065	±0.00072
14	±0.00041	±0.00032

Numbers listed in parentheses are the measured values.
The agreement between the parenthesized and the corresponding unparenthesized numbers for $n \leqslant 5$ further strengthens the validity of this assumption and scaling procedure. Comparing these values with the corresponding values for Energy Saver dipoles we can see that the field quality is indeed much better for Isabelle dipoles. In fact, the standard deviations of a_n and b_n fall off so slowly with increasing n for the Energy Saver that one would be well advised to make a very careful study of the effects on beam dynamics of the aggregate of these high order multipole fields to see whether they are tolerable.

IV. It is perhaps advisable to re-emphasize the very important feature observed at the beginning of the paper.

"Having achieved the highest obtainable accuracy in conductor placement, the only way to further improve the field quality relevant to beam dynamics is to increase the coil aperture."

Appendix (Added September 4, 1979)

Measurement data from the Westinghouse Isabelle dipoles B0001 to B0006 are now available and are given in the following table. This table further strengthens all conclusions I to VI.

Isabelle Dipoles (5 dipoles B0001-B0005)

(r = 6.55 cm = 2.58 in) (b_2 and b_4 also include B0006)

	Field quality			
n	$a_n(10^{-4} in^{-n})$		$b_2(10^{-4} in^{-n})$	
1	−0.11±2.3	(±1.8)	−0.70±1.2	(±1.8)
2	−0.25±0.58	(±0.45)	−34.9±2.1	(−17.3±0.45)
3	0.22±0.43	(±0.20)	0.064±0.16	(±0.20)
4	−0.0074±0.053	(±0.083)	1.42±0.17	(1.59±0.083)
5	−0.0001±0.008	(±0.034)	−0.003 ±0.012	(±0.034)

	Multipole current I_n/I_o	
n	$r^n a_n(10^{-4})$	$r^n b_n(10^{-4})$
1	−0.29±5.90	−1.81±3.08
2	−1.66±3.86	−232±14
3	3.85±7.39	1.10±2.75
4	−0.33±2.36	62.7±7.3
5	−0.01±0.87	−0.29±1.33

(Numbers in parentheses are design values and tolerances.)